Ph.D. dissertation

Static and dynamical properties of spin-1 Bose gas in a magnetic field

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Theses

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Introduction

Bose system is a quantummechanical many-body system containing particles with spin values of whole number in units of Planck constant ($\hbar$). Quantumstatistics of this type of systems are named Bose–Einstein statistics after their founders. This statistics carries the possibility of phase transition for particles with non-zero mass. This phase transition, called Bose–Einstein condensation, takes place at a very low temperature and below the transition temperature a finite fraction of atoms is in the lowest energy states.

Existence of Bose–Einstein condensation was debated in the early days and there was no possibility to prove its existence by experiments because of technical conditions. An open question remained how the supposed phase transition in the ideal quantum gas was influenced by an interaction between atoms.

After that the general theory of second order phase transition was worked out (L. D. Landau), it has become evident that behind development of superfluidity of ^4He a phase transition lies and Bose–Einstein condensation has been predicted in ^4He at low temperature. Connection between microscopic and Landau's phenomenological models of Bose systems was investigated by Bogoliubov and Beliaev. Their theory contains the existence of Bose–Einstein condensation fundamentally.

Since the midst of 1980's there had been a vigorous activity to achieve of Bose–Einstein condensate. Using known and newfound cryogenic techniques lower and lower temperatures were reached, however, the needed temperatures about and below μK proved to be a very difficult challenge. At last in 1995 two groups independently produced Bose–Einstein condensate verifiable. One of them was the MIT (Massachuttes Institute of Technology) group led by Wolfgang Ketterle and the other one was the JILA (Joint Institute for Laboratory Astrophysics) group led by the young Eric. A. Cornell and Carl E. Wieman, they produced condensate in ^{23}Na and ^{87}Rb gases, respectively.

First condensates were produced by using the so called magnetic trap in which spin degrees of freedom of atoms are fixed. This type of condensate is called scalar condensate, since the condensate can be characterized by a scalar wave function. Later has arisen a possibility of moving condensates from magnetic traps to a so called optical traps. In this type of traps spin degrees of freedom are not fixed and that allowed to examine more colorful behaviour of Bose–Einstein condensation.

Basic equation of scalar Bose systems is the Gross-Pitaevskii equation. It can be derived using standard quantummechanical considerations as well as second quantized formalism applied to many-body systems. This is originally a zero temperature approximation, which gives the ground state, but it can be generalized to provide information regarding the non-zero temperature behaviour as well.

Description of Bose systems, taking into account interaction between particles, is a complicated problem and cannot be solved exactly. Low temperature behaviour of a many-body system can be featured well enough by the quasiparticle picture. The idea behind is that low-lying energy states of a complicated system can be obtained as the total energy of an ideal gas. This ideal gas is a system of quasiparticles with energy spectra $\epsilon(\mathbf{k}) = \hbar\omega(\mathbf{k})$. One way to determine the energy spectra of quasiparticles is applying linear response theory, which gives spectra of collective excitations. It is based on determining correlation functions connected to invariant quantities and by investigating their singularities one can obtain spectra of collective excitations. Usually these are complex expressions, real parts and imaginary parts give energies and damping of excitations, respectively. In Bose gases particle density and magnetization are invariant quantities. In case of scalar gas with fixed spin degree of freedom one can observe particle density wave only, in spinor gas, however, different spin wave excitations also arise. Applying magnetic field will complicate behaviour of excitations.

Depending on the type of interaction between particles in a Bose gas there can be two different types of behaviour. If the sign of coupling constant of spin dependent interaction between spins is positive (negative), we speak about polar (ferromagnetic) case, respectively. Bose condensation is different in the two cases, there is different behaviour in excitations, the equation of state will also be different.

Applied methods

- The Hamilton operator was written up in second quantized form in a homogeneous magnetic field and Bose–Einstein condensation was treated by a canonical transformation.
- Green- and correlation functions were determined by applying the perturbation theory to this Hamilton operator.
- One particle and collective excitations were calculated from analytical continuation to the upper half complex plane of Green- and correlation functions.
- Analytical and numerical methods were applied to obtain spectra of elementary excitations and the equation of state was solved numerically.

Theses

I summarize the most important results in the following points:

I. The Hugenholtz–Pines theorem was generalized to spinor condensates in magnetic field.

II. A sum rule was derived to the susceptibility at constant density.

III. In the polar case there are P2 and P1 phases in $B-T$ phase space with two and one nonzero spinor condensate components, respectively.

IV. It was shown in P2 and P1 phases of the polar case that the number of Goldstone modes is two and one, respectively.

V. It was shown that the separation line between phases P2 and P1 ends in a quantum phase transition point as the temperature goes to zero and the crossover is characterized by critical exponents.

VI. It was shown in the ferromagnetic case that at a constant density a magnetic transition can occur before the Bose–Einstein condensation and the extension of magnetic phase was investigated as a function of magnetic field and strength of spin dependent interaction.

VII. Properties of soft mode related to the critical point of first order magnetic transition were determined.

Publication list

Ph.D. dissertation based on the following articles

- Krisztián Kis-Szabó, Péter Szépfalusy, and Gergely Szirmai. *Static properties and spin dynamics of the ferromagnetic spin-1 Bose gas in a magnetic field.* Phys. Rev. A, **72**, 023617 (2005)

- Gergyely Szirmai, Krisztián Kis-Szabó, and Péter Szépfalusy. *Phase separation of ferromagnetic spin-1 Bose gases in non-zero magnetic field.* Eur. Phys. J. D, **36**, 281-287 (2005)

- Krisztián Kis-Szabó, Péter Szépfalusy, and Gergely Szirmai. *Phases of a polar spin-1 Bose gas in a magnetic field.* Phys. Lett. A, **364**, 362-367 (2007)

Other publications related to Ph.D. dissertation

- Gergely Szirmai and Péter Szepfalusy and Krisztán Kis-Szabo. *Energies and damping rates of elementary excitations in spin-1 Bose-Einstein-condensed gases.* Phys. Rev. A, *68*, 023612 (2003)

Other publications not related to Ph.D. dissertation

- Bakonyi I, Péter L, Rolik Z, Kis-Szabó K, Kupay Z, Tóth J, Kiss LF, Pádár J. *Decomposition of the giant magnetoresistance of multilayers into ferromagnetic and superparamagnetic contributions.* Phys Rev B, *70*, 054427 (2004)